Causal vs. analytic constraints on anomalous quartic gauge couplings

L. Vecchi

INFN, Sezione di Trieste, and
Scuola Internazionale Superiore di Studi Avanzati,
via Beirut 4, I-34014 Trieste, Italy
E-mail: vecchi@sissa.it

ABSTRACT: We derive one loop constraints on the anomalous quartic gauge couplings using a general non-forward dispersion relation for the elastic scattering amplitude of two longitudinally polarized vector bosons. We show that for exactly chiral theories more stringent bounds can be obtained by the assumption that the underlying theory satisfies the causality principle of Special Relativity.

Contents

1. Introduction

The general structure of an effective lagrangian is dictated by the interplay between quantum mechanics, Poincaré invariance, and internal symmetries. Its coefficients are not constrained by the symmetries and must be determined by experiments. Unitarity usually sets an upper bound on the energy scale below which a perturbative effective approach is reliable.

We can interpret the standard model (SM) as an effective theory extending its lagrangian to include new non-renormalizable operators with unknown coefficients. Some of them enter the scattering amplitudes of longitudinally polarized vector bosons. These are called anomalous quartic gauge couplings since they measure the deviation from the SM predictions. These coefficients are necessarily connected with the not yet observed Higgs sector. In the case the Higgs boson is not a fundamental state, or even no Higgs boson will be observed, they provide important informations on the nature of the electro-weak symmetry breaking sector. Whereas there are no significant experimental bounds on them at the moment [1], theoretical arguments can reduce significantly their allowed range and can serve as a guide for future experiments.

The authors of [2] have noticed that the coefficients of a general effective lagrangian may be constrained by requiring the S-matrix of the full theory respects some desirable property such as analyticity, crossing symmetry, Lorentz invariance and unitarity.

We follow these authors and consider the SM $SU(2) \times U(1) \to U(1)$ breaking pattern in the case there exists a light Higgs-like boson as well as in the case no Higgs boson can propagate under the cut off of the effective theory. We show that a general non-forward dispersion relation leads to a less constraining bound than the one derived by the request the UV completion respects the causality principle of Special Relativity. This is not surprising because it is commonly believed that the analytical properties of the S-matrix are a consequence of its causal nature.

2. Analytical bounds

We briefly review an analytic tool which has been used in the context of the chiral lagrangian of QCD to constrain some effective coefficients.

Consider a multiplet of scalar particles, which to be definite we call pions π^a, having mass m. Assume they are lighter than any other quanta and that they have appropriate quantum numbers to forbid the transition $2\pi \to \pi$. The other states can be general unstable quanta of complex masses M much greater than $2m$. The S-matrix element for a general transition $2\pi \to 2\pi$ is a Lorentz scalar function of the Mandelstam variables s, t, u and of the mass m^2.

We study the amplitude for the elastic scattering $\pi^a \pi^b \to \pi^a \pi^b$ and assume it can be analytically continued to the complex variables s, t. We denote this analytical function by $F(s,t)$ and require that its domain of analyticity be dictated entirely by the optical theorem and the crossing symmetry. More precisely, we assume that the singularities come from simple poles in the correspondence of the physical masses of the quantum states which can be produced in the reaction, and branch cuts in the real axis starting at the threshold of multi-particle production.

Since no mass-less particle exchange is included in $F(s,t)$, the analytical amplitude satisfies a twice subtracted dispersion relation for a variety of complex t [3]. For any non singular complex point s, t we can write:

$$\frac{1}{2}\frac{d^2 F(s,t)}{ds^2} + P = \int_{4m^2}^{\infty} \frac{dx}{\pi} \left\{ \frac{ImF(x+i\varepsilon,t)}{(x-s)^3} + \frac{ImF_u(x+i\varepsilon,t)}{(x-u)^3} \right\} \tag{2.1}$$

where we defined $u = 4m^2 - s - t$ and used the crossing symmetry to write the amplitude in the u-channel as $F_u(x,t) = F(4m^2 - x - t, t)$.

The P on the left hand side of (2.1) denotes the second derivative of the residues. By the analyticity assumption this term comes entirely from the complex simple poles produced by the exchange of unstable states. In our discussion the pole term can be neglected since its contribution turns out not to be relevant .

In the case of forward scattering ($t = 0$) the imaginary part $ImF(x,0)$ is proportional to the total cross section of the transition $2\pi \to$ 'everything' and is therefore non negative. The crossing symmetry leads to a similar result for the u-channel. We conclude that $F''(s,0)$ is a strictly positive function for any real center of mass energy s in the range $0 \le s \le 4m^2$.

The analyticity assumption can be used to generalize the domain of positivity of the imaginary part of the amplitude. This can be seen by expanding $ImF(x+i\varepsilon,t)$ in partial waves in the physical region and observing that, due to the optical theorem and the properties of the Legendre polynomials, any derivative with respect to t at the point $x \ge 4m^2$, $t = 0$ is non negative. The Taylor series of $ImF(x+i\varepsilon,t)$ for $t \ge 0$ is therefore greater than zero. Since an analog result holds for the u-channel, we conclude that the second derivative $F''(s,t)$ is strictly positive (and analytical) for any real kinematical invariant belonging to the triangle $\Delta = \left\{ s, t, u \mid 0 \le s, t, u \le 4m^2 \right\}$.

In QCD, the scattering of pions at a scale comparable with their masses is very well described by the chiral lagrangian. The 4 pion operators produce order s^2 corrections to

the scattering amplitude and eq. (2.1) implies positive bounds on some combination of their coefficients (see [4], for example).

2.1 Application to the gauged chiral lagrangian

We can think of the SM as an effective theory and extend its action to include non renormalizable operators in the standard way [5].

The anomalous quartic gauge couplings enter the scattering amplitude of two longitudinally polarized gauge bosons at order s^2. We expect that the method outlined in the previous section may be used to bound these coefficients.

There exists, however, a fundamental difference from the QCD case. The assumptions made to derive the relation (2.1) are the analytic, Lorentz and crossing symmetric nature together with the asymptotic behavior of the amplitude $F(s, t)$. A sufficient condition for the latter hypothesis to hold is that no massless particle exchange contribute to F (Froissart bound). In the electroweak case this latter assumption is not natural because of the presence of the electromagnetic interactions.

Although we may consider only amplitudes with no single photon exchange (like $W^{\pm}Z^0 \rightarrow W^{\pm}Z^0$ for example), there is still an operative difficulty due to the fact that the amplitude F is generally dominated by the SM graphs at low energy scales. These latter give rise to positive contributions to $F(s, t)$, since the SM is well defined even for vanishing coefficients, and one is lead to conclude that eq. (2.1) implies that the effective operators involved cannot produce a "too large and negative" contribution to the amplitude $F(s, t)$ and that, as a consequence, no significant bound can be derived in the gauged theory. Notice that this is also true in the absence of a light Higgs boson as far as the CM energy is of the order of the Z^0 mass.

One way to overcome these apparent complications is considering amplitudes with no single photon exchange and evaluating them at a high scale $s \gg m_Z^2$ with the equivalence theorem (ET). In this case one has to prove the positivity of the second derivative of the amplitude is guaranteed in the energy regime in which the approach is defined [6].

Another way, which we decide to follow, is working in the global limit. The crucial observation in order to justify this assumption is that in the matching between the effective lagrangian and the UV theory the transverse gauge bosons contribute, because of their weak coupling, in a subdominant way to the effective coefficients of our interest. An accurate estimate of them, and the respective bounds, can therefore be obtained neglecting completely the gauge structure and studying the coefficients of the global theory.

Using this conceptually different (though operationally equivalent) perspective we can study any two by two elastic scattering amplitude and generalize the analysis of [6] to non-forward scattering.

2.2 Derivation of the analytical bounds

We first specialize to the case there appears no Higgs-like boson under a cut off Λ.

In this context the basic tool is a non linearly realized effective lagrangian for the breaking pattern $\mathrm{SU}(2) \times \mathrm{U}(1) \rightarrow \mathrm{U}(1)$ written in terms of a $\mathrm{SU}(2)$ matrix $U = \exp(i\pi^a \sigma^a / v)$, where σ^a are the three Pauli matrices with $a = 1, 2, 3$ and $v \simeq 250$ GeV is the EW vacuum.

As usual, under a global $SU(2)_L \times U(1)_Y$ transformation $U \to LUR^\dagger$, where $L \in SU(2)_L$ and $R \in U(1)_Y \subset SU(2)_R$.

Assuming $m_Z^2 \ll \Lambda^2$ and working at energies comparable with the Z^0 mass, the most general lagrangian respecting the above symmetries and up to $O(s^2)$ is given in reference [7]. The globally symmetric version is:

$$
\begin{aligned}
\mathcal{L}_{\text{EWChL}} = & -\frac{v^2}{4}\text{Tr}\,(V_\mu V^\mu) + \frac{1}{4}\beta_1 g^2 v^2 [Tr(TV_\mu)]^2 \\
& + \alpha_4 [Tr(V_\mu V_\nu)]^2 + \alpha_5 [Tr(V_\mu V^\mu)]^2 + \alpha_6 Tr(V_\mu V_\nu) Tr(TV^\mu) Tr(TV^\nu) \\
& + \alpha_7 Tr(V_\mu V^\mu) Tr(TV_\nu) Tr(TV^\nu) + \frac{1}{2}\alpha_{10}[Tr(TV_\mu) Tr(TV_\nu)]^2,
\end{aligned} \tag{2.2}
$$

where $V_\mu = (\partial_\mu U)U^\dagger$ and $T = U\sigma^3 U^\dagger$.

We stress that in this idealized scenario the π^a are exact Goldstone bosons. To avoid any complication with the asymptotic behavior of the amplitude we can introduce by hand a π^a mass and proceed as in QCD. This mass is actually the consequence of an explicit symmetry breaking term in the UV theory. Being interested in constraining the underlying symmetric theory we are forced to take $m^2 \ll m_Z^2, s$. The bounds we derive differ from the QCD ones for this very reason.

Although no mass gap is present in this context, an approximate positive constraint for $F''(s,t)$ can be derived. This we do by noticing that a general dispersion relation like (2.1) can be used to bound the anomalous quartic couplings only if the $O(s^3)$ contribution to $F(s,t)$ is negligible. In this regime the second derivative $F''(s,t)$ is dominantly s independent and, for a small non vanishing imaginary part for s, the dispersion relation can be approximated as:

$$
\frac{1}{2}\frac{d^2 F(s,t)}{ds^2} \simeq \int_0^\infty \frac{dx}{\pi} \left\{ \frac{ImF(x+i\varepsilon,t)}{x^3} + \frac{ImF_u(x+i\varepsilon,t)}{x^3} \right\} \left(1 + O\left(\frac{s,t}{\Lambda^2}\right) \right) \tag{2.3}
$$

where the limit $m^2/s \to 0$ was assumed and the resonant pole term has been neglected. Eq. (2.3) shows that, as far as $O(s^3)$ are negligible compared to $O(s^2)$, the second derivative of the amplitude is strictly positive.

Before evaluating the bounds we notice that the smallness of the EW precision tests T parameter [8] is conveniently achieved by assuming the existence of an approximate global $SU(2)_C$ custodial symmetry under which the Goldstone boson matrix transforms as the adjoint representation. The dominant coefficients associated to anomalous quartic gauge operators are α_4 and α_5 and any $\pi^a\pi^b \to \pi^c\pi^d$ scattering amplitude can be written in terms of a function $A(s,t,u)$. The relevant processes turn out to be:

$$
\begin{aligned}
A(\pi^0\pi^0 \longrightarrow \pi^0\pi^0) &= A(s,t,u) + A(t,s,u) + A(u,t,s) \\
A(\pi^\pm\pi^0 \longrightarrow \pi^\pm\pi^0) &= A(t,s,u),
\end{aligned} \tag{2.4}
$$

where, at one loop level and in the limit $m^2/s \to 0$, we have [9]

$$
\begin{aligned}
A(s,t,u) = & \frac{s}{v^2} + \frac{4}{v^4}\left[2\alpha_5(\mu)s^2 + \alpha_4(\mu)(t^2+u^2) + \frac{1}{(4\pi)^2}\frac{10s^2 + 13(t^2+u^2)}{72} \right] \\
& - \frac{1}{96\pi^2 v^4}\left[t(t-u)\log\left(-\frac{t}{\mu^2}\right) + u(u-t)\log\left(-\frac{u}{\mu^2}\right) + 3s^2\log\left(-\frac{s}{\mu^2}\right) \right]
\end{aligned} \tag{2.5}
$$

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Notice that we have chosen to work with the renormalized coefficients $\alpha_{4,5}(\mu)$ as defined by the modified minimal subtraction scheme, rather than using the non standard normalization of [9].

We can now derive (2.4) twice with respect to s and evaluate the result at $s + i\varepsilon, t$, where $0 < s, t \ll \Lambda^2$. It is convenient to choose a different representation for the kinematical invariants in order to eliminate the logarithms in the final result. We define a scale $w = \sqrt{s(s+t)} = \sqrt{-su} > s$ and obtain:

$$\alpha_4(w) + \alpha_5(w) > -\frac{1}{16}\frac{1}{(4\pi)^2}$$
$$\alpha_4(w) > \frac{1}{12}\frac{1}{(4\pi)^2}\left(-\frac{7}{6} + \frac{1}{8}\left(\frac{w}{s} + \frac{s}{w}\right)^2\right). \tag{2.6}$$

For $t = 0$ we have $\alpha_4 + \alpha_5 \gtrsim -0.40 \times 10^{-3}$ and $\alpha_4 \gtrsim -0.35 \times 10^{-3}$ at an arbitrary scale $w = s \ll \Lambda^2$. This result coincides with the one obtained in [6], as expected.

In the case of non-forward scattering, the bound on $\alpha_4(w)$ cannot get arbitrarily large (large w or, equivalently, large t) because at some unknown scale, much smaller than Λ^2, the $O(s^3)$ corrections become relevant in the determination of the amplitude and the bound would not apply. Without a detailed knowledge of the perturbative expansion in the weak coupling s/Λ^2, (that is, of the full theory!) we cannot realistically tell which is the strongest bound derived by this analysis.

What we can certainly do is to compare (2.6) with the well known constraints on the corresponding parameters $l_1 = 4\alpha_5$ and $l_2 = 4\alpha_4$ of QCD. Strong bounds on these coefficients have been evaluated in the triangle Δ [10]. We may interpret our analysis as a study of the axiomatic constraints on the two pion amplitudes in the complementary region $m^2 \ll s \ll \Lambda^2$. Using the notation introduced in [9] we translate (2.6) into $2\bar{l}_1 + 4\bar{l}_4 \gtrsim 3$ and $\bar{l}_2 \gtrsim 0.3$. These constraints are compatible with the experimental observations [11] but are less stringent than those obtained in [10].

We conclude that our analysis does not lead to an improvement of the bounds on $\bar{l}_{1,2}$. If the chiral symmetry is exact, on the other hand, eqs. (2.6) represent stringent bounds on the anomalous quartic couplings implied by the assumptions of analyticity, crossing symmetry, unitarity and Lorentz invariance of the S-matrix.

Eq. (2.3) is not rigorous if a light state enters the processes under consideration and therefore (2.6) are not valid if a Higgs-like scalar propagates under the cutoff. In the next paragraph we discuss an approach which works in this context as well, provided the chiral symmetry is exact.

3. Causal bounds

Given a general solution of the equations of motion derived from (2.2) we can study the oscillations around it. Consistency with Special Relativity requires the oscillations to propagate sub-luminally. This request may be expressed as a constraint on the same coefficients which enter the elastic scattering of two Goldstone bosons because the dynamics of the oscillation on the background can be interpreted as a scattering process on a macroscopic

'object'. If the background has a constant gradient, the presence of super-luminal propagations sum up and can in principle become manifest in the low energy regime [2].

A constant gradient solutions admitted by the lagrangian (2.2) is defined by $\pi_0 = \sigma C_\mu x^\mu$, where σ is a generic isospin direction and the constant vector C_μ is fine-tuned in order to satisfy $C^2 \ll v^4$. The quadratic lagrangian for the oscillations $\delta\pi = \pi - \pi_0$ around the background have the general form:

$$\mathcal{L} = \delta\pi \left(p^2 + \frac{\alpha}{v^4} (Cp)^2 \right) \delta\pi, \tag{3.1}$$

with $\alpha = \alpha_4, \alpha_4 + \alpha_5$. In the evaluation of (3.1) we neglected $O(Cx/v)$ terms. We can imagine in fact the non trivial background to be switched on in a finite space-time domain so that the latter approximation is seen as a consequence of the fine-tuning of the parameter C_μ.

A perturbative study of the interacting field $\delta\pi$ is in principle possible for energies under a certain scale (to be definite we call this scale the cut-off of the effective theory). By assumption, this cut off is arbitrarily close to Λ as C^2/v^4 goes to zero and, having this fact in mind, we simply denote it as Λ.

A necessary condition for such a perturbative study to make any sense is that the quadratic lagrangian be well defined. This is the case for (3.1) only if $\alpha \geq 0$. In fact, the field $\delta\pi$ has velocity $dE/dp = E/p$ (where $p^\mu = (E, \bar{p})$ and $|\bar{p}| = p$) and for $\alpha < 0$ its quanta propagate super-luminally.

It is important to notice that the presence of super-luminal modes is not the consequence of a bad choice of the vacuum. The quadratic hamiltonian is stable in any vacuum (parametrized by C_μ) if α is 'sufficiently small' but generally leads to violations of the causality principle of Special Relativity when $\alpha < 0$. In the latter hypothesis then different inertial frames may not agree on the physical observations and, for example, the quadratic hamiltonian may appear unbounded from below to a general Lorentzian frame boosted with a sufficiently high velocity.

We finally interpret the constraint $\alpha \geq 0$ as a causal bound.

The effective coefficients α which appear in the perturbative analysis are actually the renormalized couplings so that the above bound can be extended to all energy scales $w < \Lambda^2$, where the perturbative study is assumed to be meaningful, after taking into account the running effect:

$$\alpha_4(w) + \alpha_5(w) \geq \frac{1}{8} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{w}\right)$$
$$\alpha_4(w) \geq \frac{1}{12} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{w}\right). \tag{3.2}$$

This approach may be applied even to scenarios in which a scalar Higgs, composite or fundamental, can propagate under the cut off. In this latter case the causal constraints read $\alpha_4 \geq 0$ and $\alpha_4 + \alpha_5 \geq 0$ but now the coefficients do not have any scale dependence because the theory has no extra-SM divergences at order s^2. Therefore, the possibility $\alpha_4 = \alpha_5 = 0$ can not and must not be excluded (consider the particular example of the SM). The analytical bounds, which would imply a strict inequality, do not apply as already noticed.

The bounds 3.2 cannot be compared to the QCD ones because π_0 does not solve the equations of motion when $m \neq 0$.

4. Conclusions

We have derived general bounds on the anomalous quartic gauge couplings using two distinct approaches. The causal one relies on the absence of superluminal propagations. The analytical one relies on the assumption of analyticity, crossing and Lorentz symmetry together with a good behavior at infinity of the scattering amplitude $F(s,t)$. The latter method works in the context of a strongly coupled theory with no Higgs propagating at low energy only. In this scenario (2.6) can be compared to (3.2). We see that the bound on $\alpha_4 + \alpha_5$ is clearly dominated by the causal result and that this is also the case for α_4 if, roughly, the ratio $(w/s)^2$ does not exceed $16 \log(\Lambda/\sqrt{w})$. We cannot tell if the analytical bound still apply up to this scale

More importantly, if the fermionic effects are considered separately from $\alpha_{4,5}$, a realistic estimate of the constraints should take the fermions couplings to the Goldstone bosons into account. It is easy to see that the one loop effect induced by the SM fermions gives rise to a positive contribution to the second derivative of the amplitude. This of course lowers the analytical bounds while the causal argument remains valid and (3.2) is not altered.

The bound (3.2) for the higgsless scenario, together with the constraint $\alpha_4 \geq 0$ and $\alpha_4 + \alpha_5 \geq 0$ for the light Higgs-like scenario provide the most stringent and reliable bounds on the effective coefficients $\alpha_{4,5}$.

In order to have a rough estimate of (3.2) we assume $\Lambda \sim 1\,\mathrm{TeV}$ and get $\alpha_4 + \alpha_5 \gtrsim 3.8 \times 10^{-3}$, $\alpha_4 \gtrsim 2.5 \times 10^{-3}$ at the Z^0 pole. These values lie inside the very wide experimental bounds $-0.1 \lesssim \alpha_{4,5} \lesssim 0.1$. Eqs. (3.2) significantly reduce the allowed range.

The experimental constraints are extremely weak since they have been derived by estimating the loop corrections induced by $\alpha_{4,5}$ on the electroweak precision parameters [1]. A direct measurement of the anomalous gauge couplings turns out to be of fundamental importance in order to have some insight on the actual nature of the electroweak breaking sector [13]. LHC may improve the bounds [1] by an order of magnitude but the linear collider seems far more appropriate to resolve the coefficients [12]. The measurement of a negative value of α_4 and $\alpha_4 + \alpha_5$ at the next linear collider would therefore signal a breaking of causality, irrespective of the presence of a light scalar state like the Higgs boson. This seems a rather unlikely possibility because it would require too drastic a modification of our physical understanding. A more conservative point of view consists in interpreting the bounds (3.2) as theoretical constraints on the full theory.

Acknowledgments

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